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THE EFFECT OF PERTURBATIONS OF CONVECTIVE ENERGY TRANSPORT ON THE LUMINOSITY AND RADIUS OF THE SUN[1]

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ABSTRACT

The response of solar models to perturbations of the efficiency of convective energy transport is studied for a number of cases. Such perturbations primarily effect the shallow superadiabatic layer of the convective envelope (at depths $\lesssim 10^3$ km below the photosphere). Independent of the details of the perturbation scheme, the resulting change in the solar radius ($\Delta R/R$) is always very small compared to the change in luminosity ($\Delta L/L$). This appears to be true for any physical mechanism of solar variability which operates in the outer layers of the convection zone.

Changes of the solar radius have been inferred by Dunham *et al.* (1980,1981) from historical observations of solar eclipses in 1715 and 1925. Considering the constraints on concurrent luminosity changes, this type of solar variability must be indicative of changes in the solar structure at substantial depths below the superadiabatic layer of the convective envelope.

Subject headings: convection - Sun: general - Sun: interior

I. INTRODUCTION

The Sun is often cited as a paradigm of constancy, both in common folklore and in a stricter astrophysical context. The theory of stellar structure and evolution supports this view by placing the Sun in the main sequence stage, where conversion of hydrogen to helium provides a stable energy source for some 10^{10} years. However, there are a number of physical mechanisms which can produce short term variations in the Sun that are not addressed within the framework of classical stellar evolution theory. Some of these mechanisms are: nonradial instabilities associated with the distribution of He^3 in the solar core (Dilke and Gough 1972), beat interactions between g-mode oscillations in the core (Wolff 1976), effects of dynamo-generated magnetic fields on the convective envelope (Thomas 1979; Spiegel and Weiss 1980), and fluctuations in energy transport through the convective envelope due to the stochastic nature of turbulent convection (Dearborn and Newman 1978). In most cases, these mechanisms are too complex to be modeled with any confidence on a purely theoretical basis. For this reason, it is advisable to look to the observations for guidance and to use *ad hoc* modeling to relate observations to basic physical mechanisms.

The observations we address in this paper concern variations of the solar luminosity and radius. We will use perturbed solar models to investigate whether these observations can be related to fluctuations in energy transport in the convective

envelope of the Sun. The pertinent observations are summarized in §II; the models are described in §III. The conclusion derived in §III is that solar models do not respond to changes in convective energy transport in the manner suggested by the observations. This suggests either that stellar evolution models are inappropriate for investigating solar variability or that some other physical mechanism is responsible for the variability. These alternatives are considered in some detail in §§IV and V, respectively.

II. A SUMMARY OF THE OBSERVATIONS

a) Irradiance Measurements

Measurements of the solar luminosity actually refer to the irradiance S (solar constant), which is the flux emitted in the direction of the Earth. Ground-based measurements were made by the Smithsonian Astrophysical Observatory from 1902 to 1962, with the most extensive data covering the period 1923 to 1954. Based on a statistical analysis of the monthly means for the latter period, Hoyt (1979) concluded that the measured variations (of order $\pm 0.5\%$) are due primarily to incomplete compensation for atmospheric effects (see, also, Roosen, Angione, and Klemcke 1973). According to Hoyt, the Smithsonian data can probably set a limit $|\Delta S/S| \leq 5 \times 10^{-3}$ for irradiance variations over the period covered by the measurements.

Because of the large corrections for atmospheric absorption (typically 25 to 40%) required in ground-based observations, modern measurements have been made at balloon altitudes or higher. Fröhlich (1977) and Fröhlich and Brusa (1981) have reviewed the balloon and rocket measurements and conclude that $|\Delta S/S| < 1.5 \times 10^{-3}$ from 1969 to 1980 (i.e., over one sunspot cycle). The subject of solar irradiance variations is often controversial and the claimed detections of variability (e.g., Kondratyev and Nikolsky 1970) must be treated with healthy skepticism.

The Nimbus 7 and Solar Maximum Mission (SMM) satellites have recently detected variations in the solar irradiance (Hickey *et al.* 1980; Willson *et al.* 1981). The reality of these

variations is confirmed by their simultaneous detection by the two independent experiments. For the two-year interval (1978-80) covered by the available date, the irradiance varied by as much as 0.2% from the mean ($|\Delta S/S| = 2 \times 10^{-3}$). These fluctuations are rapid (time scales $\lesssim$ 10 days) and upper limits on secular trends are much smaller. Perhaps more important, Willson et al. (1981) found that the major variations in the SMM measurements coincided with the passage of large sunspot groups across the central meridion of the solar disk. This suggests that the observed irradiance variations are due to local blocking of the solar radiation in the vicinity of active regions. In this case, caution must be used in relating observed irradiance variations to changes in the solar luminosity. A recent analysis of the SMM data by Oster, Schatten, and Sofia (1981) concludes that the irradiance variations are dominated by anisotropy effects (directionality of the radiation) and that true variations of the solar luminosity are smaller than one-tenth of the irradiance variations (i.e., $\Delta L/L \leq 0.1\ (\Delta S/S) \simeq 2 \times 10^{-4}$, for short-term fluctuations). In this case, observations over a long period of time will be required to detect global luminosity variations.

b) Radius Measurements

Solar radius changes of sufficient amplitude would show up in the transit measurements made at a number of national observatories. The measurements made at the Greenwich Royal Observatory between 1850 and 1937 have been analysed by Sofia et al. (1979). They found that $|\Delta R/R| < 2.5 \times 10^{-4}$ for any secular

trend during this period.[1] Shapiro (1980) used timings of the

[1]Eddy and Boornazian (1979) also used the Greenwhich data to look for trends in the solar radius and claimed a steady decrease amounting to $\Delta R/R = -1 \times 10^{-3}$ for the period 1836-1953. However, this large rate of shrinkage is ruled out by a number of other measurements (described in the text).

passage of Mercury across the solar disk to establish an independent limit on long-term changes in the solar radius. For the period 1736-1973, Shapiro found $|\Delta R/R| < 3.6 \times 10^{-4}$ for any linear trend. It should be noted, however, that both the transit data and the Mercury data show large amounts of scatter so nonsecular variations of the radius could be much larger than the limits given above. Gilliland (1981) has reanalyzed transit measurements and the Mercury data. By combining several data sets, he found marginal (2σ) evidence for a 76-year periodicity, with an amplitude $\Delta R/R \sim 2 \times 10^{-4}$.

The solar radius can be very accurately determined from observations of the width of the path of totality in a solar eclipse. Unfortunately, the required data (eclipse duration near the edge of the path of totality) are only available for a few eclipses. The inferred radius changes, relative to a standard reference value, are listed in Table 1. It should be noted that the uncertainty in the 1715 value refers to an estimate of the <u>maximum</u> possible error since insufficient data are available to form a statistical error estimate (which would be significantly smaller). The 1715 and 1925 eclipses give radius values which differ by $\Delta R/R \simeq 6 \times 10^{-4}$ from recent values. The

difference between the 1925 and 1980 values is highly significant (greater than 5σ).

Considering the dates of the observations, it is apparent that there is no linear trend, and the changes may well be stochastic. The 1715, 1925, and 1976 measurements all occur on the rising branch of the sunspot cycle so there is no obvious correlation with solar activity. Also, according to the 76-year periodicity claimed by Gilliland (1981), the 1925 and 1980 eclipses should have occurred near maximum solar radius and the 1715 eclipse should have fallen near minimum radius. This is clearly in conflict with the data, indicating that Gilliland's suggested periodicity is not real. The question of periodicity, and time-dependent behavior in general, can only be addressed when additional observations become available. At this point, it is only possible to determine that radius changes $\Delta R/R \simeq 6 \times 10^{-4}$ occur on some time scale $t \lesssim 50$ years.

c) Photospheric Velocity Fields

Solar radius changes ΔR, on some time scale t, imply expansion velocities $V \simeq \Delta R/t$, which would cause a variable center-to-limb Doppler shift across the solar disk. If we assume that the Sun is presently in a variable state, then modern Doppler measurements can be used to set upper limits on $\Delta R/t$.

Figure 1 shows the resulting limits on the time scale of solar radius variations. The limit of spectroscopic detection has been set at 1 m/s, on the basis that large-scale variable velocity fields as small as 2 to 3 m/s have been measured (Severny, Kotov, and Tsap 1976; Howard and LaBonte 1980). This

detection limit sets a lower limit of ~5 days for the time scale of <u>recent</u> radius changes of order $\Delta R/R = 6 \times 10^{-4}$. Thus, reasonable (though not certain) limits on the time scales of the radius changes are: 5 days $\lesssim$ t $\lesssim$ 50 years. While not very useful from an observational viewpoint, these limits are important in defining the modeling regime (see §§III and IV).

Figure 1 also shows the approximate locations in the V-t plane of the 5-minute and 160-minute solar oscillations. These oscillations have amplitudes which are factors of 60 or more smaller than the radius changes we are considering in this investigation.

III. PERTURBED SOLAR MODELS

Ulrich (1975) introduced the notion of modeling solar variability by changing the characteristic length scale in the mixing length formalism for convection. Since this length scale is usually specified by α, the (constant) ratio of mixing length to the local pressure scale height, we will refer to this procedure as an α-perturbation. If an α-perturbation is introduced in a time-series of stellar evolution models, the response of the models may mimic the response of the Sun to thermal fluctuations within the convective envelope. As pointed out by Dearborn and Newman (1978), changing the mixing length is equivalent to changing the efficiency of convective heat transport.

The response of a solar model to an α-perturbation has been described in detail by Dearborn and Blake (1980) and Sweigart (1981). Briefly, for a positive perturbation (increasing α), the efficiency of convection increases, causing a sudden increase in both L and R. The luminosity perturbation decays slowly, on a time scale of order 10^5 years. By contrast, the radius perturbation appears to decay much more rapidly. This is because the initial expansion is restricted to the outer, superadiabatic layers, while the deeper layers are slowly contracting. Because the outer layers have a short thermal relaxation time, the radius returns to its unperturbed value after a few hundred years. Thereafter, the slow contraction of the inner layers dominates and the photospheric radius is smaller than the unperturbed value.

One problem with such perturbation procedure is that it is difficult to know what constitutes a realistic amplitude for changes in α. As a result, changes of R and L of almost any size can be produced. Sofia et al. (1979) suggested characterizing a particular type of perturbation by

$$W = \frac{\Delta R/R}{\Delta L/L}, \qquad (1)$$

From the behavior of a solar model described above, it is clear that W will be time-dependent. However, during the first 50 years or so after a perturbation, the value of W does not change much from its initial value. More important, the value of W is, to first order, independent of the perturbation amplitude.

Based on model calculations, Sofia et al. (1979) estimated $W \simeq 7.5 \times 10^{-2}$ for an α-perturbation. This result was questioned by Dearborn and Blake (1980) and Gilliland (1980), who obtained much smaller values for W. A recalculation using essentially, the same stellar evolution code as used by Sofia et al. showed that the earlier calculation was, in fact, in error (Twigg and Endal 1981). Values of W resulting from α-perturbations have been calculated by Dearborn and Blake (1980), Gilliland (1980), Sweigart (1981), and Twigg and Endal (1981), using independent stellar evolution codes. The reported values (immediately after a perturbation) range from 6×10^{-3} to 6×10^{-4}. Although the range of these values shows that the results are fairly code-dependent, a general conclusion that $W \ll 1$, for an α-perturbation, is warranted. We can compare this result with the value of W implied by the observations described in §II.

The eclipse measurements of the solar radius (§II.b) show that solar radius changes of order $\Delta R/R = 6 \times 10^{-4}$ have occurred within this century. On the other hand, the programs for detecting solar irradiance changes (§II.a) have failed to detect any changes. These programs cover time scales ranging from 60 years down to a fraction of a day. The observations require $|\Delta S/S| \lesssim 5 \times 10^{-3}$. Based on the expectation that $|\Delta L/L| \leq |\Delta S/S|$, this gives

$$|W| \gtrsim 0.1 \qquad (2)$$

Thus, the observations require that the magnitude of W be significantly greater than the values obtained from the theoretical calculations. This suggests that either: (a) although the observed radius changes may be due to changes in efficiency of convection, the real variability cannot be modeled as a simple linear perturbation; (b) the approximations made in stellar evolution calculations (hydrostatic equilibrium, mixing length theory of convection, etc.) are not appropriate for this type of modeling; or (c) the observed radius variations are not associated with changes in the efficiency of convection in the solar envelope. The first possibility can be tested by using models to estimate the importance of nonlinear behavior, as described below. Discussions of (b) and (c) will be deferred to §§IV and V, respectively.

If the solar radius and luminosity respond linearly to an α-perturbation, W will be independent of the amplitude of the perturbation. In this case, the value of W will not depend on the amplitude or on the time interval between successive perturbations (since the cumulative changes ΔR and ΔL add linearly). However,

the thermal structure of the Sun is governed by nonlinear equations and is characterized by a broad range of relaxation times. Under these conditions, it is not obvious that all possible values of W are obtained by the single, small-amplitude perturbations used in previous calculations. More complex perturbation schemes (perhaps involving α-perturbations of models which are not in thermal equilibrium) might give rise to larger values of $|W|$, more in accord with the observations. Such perturbation schemes may also give a better representation of the real situation, in which the Sun is never in true thermal equilibrium.

To test this possibility, we calculated the response of a solar model using various schemes for perturbing the mixing length in the convective envelope. The calculations were started from a 1 $M_{\odot}$ model evolved to an age of 4.7×10^9 yr, with the input physics described by Endal and Sofia (1981). To minimize numerical noise, a large number (700) of interior zones was used and the convergence criteria for all internal iterations were tightened by several orders of magnitude beyond normal stellar evolution values. In addition, the difference equations were rewritten in centered ("conservative") form to increase numerical stability at short time steps. The outer boundary conditions were provided by a fine grid of static envelopes of mass 6×10^{-8} $M_{\odot}$. The thermal relaxation time (1.4 years) of this mass effectively limited the time resolution of the calculations.

One way to test for nonlinear effects is to test for amplitude-dependence of W. Figure 2 shows the value of W for perturbations of various amplitudes imposed on an equilibrium model. The solid line refers to values obtained using a standard mixing

length formalism, with the mixing length proportional to the pressure scale height. The dashed line refers to an alternative formulation, which will be discussed in §IV.d. In both cases, W varies by ~ 10% over a factor of 40 range in perturbation amplitude, indicating nearly linear behavior. This is consistent with the linear relationship between $\Delta\alpha$ and ΔL found by Dearborn and Newman (1978). The smooth behavior in Figure 2 indicates that numerical noise was under control in the calculations.

We can also test for nonlinear behavior by imposing a series of perturbations on an evolutionary sequence of models (with short time steps). If the time intervals between successive perturbations are shorter than the relaxation time of some part of the model, then the perturbations may add in a nonlinear fashion since they are applied to models which are not in thermal equilibrium. Although this effect could be examined using a purely random or arbitrary sequence of α-perturbations, it seems more instructive to concentrate on some known feature of solar variability, such as the 22-year magnetic cycle. For this purpose, a 1%-amplitude, 22-year cycle of α-perturbations was imposed on a sequence of models spaced at one-year intervals.[2] The evolution was followed for 374 years (17 cycles). In order to

[2]Using time steps shorter than the 1.4-year resolution allowed by the mass of the static envelope means that the time domain was over-sampled. This does not create a problem since the time scale for the α-perturbations was much longer (22 years).

simulate the noisy character of the real solar cycle, the perturbations were generated by passing a sequence of random numbers

through a narrow bandpass filter centered at 1/22 cycle per year (see Barnes, Sargent, and Tryon 1980).

The percentage α-perturbations

$$\delta(\alpha_i) = \frac{\alpha_i - \alpha_0}{\alpha_0} \times 100 \tag{3}$$

are shown in Figure 3. Here, α_i is the mixing length to pressure scale height ratio used in model i (i = 1, ..., 374) and α_0 gives the unperturbed value. Applying the perturbations to the models yielded the following quantities of interest:

$$\delta(R_i) = \frac{R_i - R_0}{R_0} \times 100 \ , \tag{4}$$

$$\delta(L_i) = \frac{L_i - L_0}{L_0} \times 100 \ , \tag{5}$$

and

$$W_i = (\delta R_i)/(\delta L_i) \ . \tag{6}$$

These quantities are shown in Figure 4. From Figure 4(c), it is apparent that the values of W_i for multiple perturbations do not differ greatly from the value ($W \simeq 6 \times 10^{-3}$) produced in our models by a single perturbation.[3]

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[3]The W_i curves in Figures 4(c) and 5(c) show occasional sharp spikes. These spikes result from slight phase shifts of the $\delta(R_i)$ and $\delta(L_i)$ curves. Considering the limited time resolution of our calculations, these phase shifts cannot be considered real. On the other hand, the broad dips in W_i appear to be real. This dips invariably occur in the negative parts of the α-cycle and may be associated with the thermal relaxation process.

At this point, we digress slightly to consider an observational problem: the observations yield changes $\Delta R = R_i - R_j$, where neither R_i nor R_j can be identified with the equilibrium radius used as a reference in the theoretical models. To examine the effect of using different reference values to define W_i, we replaced the equilibrium values R_o and L_o in equations (4) and (5) by values from n previous time steps, i.e.,

$$\delta(\chi_i) = \left(\frac{\chi_i - \chi_{i-n}}{\chi_{i-n}}\right) \times 100 , \qquad (7)$$

where χ refers to R and L and equation (6) still defines W_i. An example of the effect of altering the reference value in this manner, for n=11, is shown is Figure 5. These values were computed from the same runs as shown in Figures 3 and 4. Although the $\delta(R_i)$ and $\delta(L_i)$ curves are now considerably more noisy, the general level of the W_i values is not substantially altered. Curves for n = 1,2,5, and 22 were also examined, giving similar results. (The n = 11 case shows the largest deviation from Figure 4.) The stability of the W values for a wide variety of perturbation schemes and operational definitions indicates that this parameter does, indeed, provide an excellent characterization of the α-perturbation.

Finally, it is possible that the α-perturbations imposed on our models were too rapid to allow nonlinear effects associated with the relaxation process to arise in the deeper parts of the convection zone. To test this, a smaller number of cycles was computed with periods of 44 and 110 years. The results confirm the conclusions given above. Time scales longer than 110 years are of doubtful relevance to the available observations.

We conclude that α-perturbations in stellar evolution models of the Sun will not yield values of W in the allowed observational range, irregardless of how these perturbations are introduced. The basic problem is that it is not possible to produce significant radius changes without introducing unreasonably large variations in the luminosity. For $W \approx 6 \times 10^{-3}$, the observed radius changes $\Delta R/R \simeq 6 \times 10^{-4}$ imply a 10% variation in luminosity. Aside from the direct observational constraints, such a large luminosity variation would produce easily detectable changes in the earth's climate (cf. Budyko 1969). For the even smaller values of W found by Gilliland (1980) and Sweigart (1981), the problem is more severe. This suggests that either the response of the Sun to changes in convective efficiency is incorrectly modeled in the calculations described above, or that some other physical mechanism is responsible for the observed radius changes.

IV. ANALYSIS OF THE MODEL APPROXIMATIONS

The approximations made in stellar evolution calculations have been thoroughly described in a number of standard texts (e.g., Schwarzschild 1958; Cox and Guili 1968). However, calculations such as those presented in §III are very different from normal stellar evolution calculations. For this reason, the important approximations, and their justifications in the present context, should be examined in some detail.

a) Spherical Symmetry

The full amplitude of the observed radius changes, $\Delta R/R \simeq 6 \times 10^{-4}$ is more than 10 times larger than the distortion of the solar disk due to the permanent oblateness, $(R_{eq} - R_{pole})/R \lesssim 5 \times 10^{-5}$ (Dicke and Goldenberg 1967; Hill and Stebbins 1975), and 30 times greater than the periodic distortion, $(R_{eq} - R_{pole})/R \simeq 2 \times 10^{-5}$, reported by Dicke (1976). Unless the oblateness measurements (in 1966 and 1973) refer to years of anomalously small distortions, we can conclude that departures from spherical symmetry do not play a role in the observed radius variations.

b) Quasi-hydrostatic Equilibrium

The usual statement in stellar evolution studies is that hydrostatic equilibrium is valid if the relevant time scales are much longer than the free-fall time:

$$t_{ff} = 2\sqrt{R^3/GM} \simeq 50 \text{ minutes, for the Sun.} \tag{8}$$

In the present context, we can quantify this statement. Any radius change must be due to an inbalance of the gravitational and pressure gradient forces. Let f be the fraction of the pressure gradient force not balanced by local gravity. Then, assuming spherical symmetry,

$$\frac{d^2R}{dt^2} = f\,\frac{GM}{R^2} = fg \ , \tag{9}$$

where g is the gravitational acceleration. For the small radius changes considered here ($\Delta R << R$), we can assume that g is essentially constant and let f denote a mean value over an expansion/contraction phase. Successive integrations of equation (9) give

$$V = \frac{dR}{dt} = fg\ t \tag{10}$$

and

$$\frac{\Delta R}{R} = \frac{1}{2}\,\frac{fg}{R}\ t^2 = 2f\left(\frac{t}{t_{ff}}\right)^2 \ , \tag{11}$$

where t is now the duration of an expansion/contraction phase. From equation (11), we obtain

$$f = \frac{1}{2}\ \frac{\Delta R}{R}\left(\frac{t}{t_{ff}}\right)^{-2} . \tag{12}$$

In principle, f can be very small and still violate the hydrostatic assumption, since strict hydrostatic equilibrium implies $f = 0$. Quasi-hydrostatic equilibrium includes the rates of change of gravitational potential energy (U_G) and internal energy, but ignores the rate of change of kinetic energy (U_k). This approximation is valid if $\dot{U}_G >> \dot{U}_k$. Using equations (9) to (11) to evaluate these rates for a unit mass at the solar surface gives

$$\dot{U}_G = \frac{d}{dt}\left(-\frac{GM}{R}\right) = \frac{GM}{R^2}\frac{dR}{dt} = fg^2\, t \tag{13}$$

and

$$\dot{U}_k = \frac{d}{dt}\left(\frac{1}{2}\, v^2\right) = v\,\frac{d^2R}{dt^2} = f^2 g^2\, t\ . \tag{14}$$

With equation (12) and $\Delta R/R \simeq 6 \times 10^{-4}$, the ratio of these rates becomes

$$\frac{\dot{U}_G}{\dot{U}_k} = \frac{1}{f} = 2\left(\frac{\Delta R}{R}\right)^{-1}\left(\frac{t}{t_{ff}}\right)^2 \simeq 3 \times 10^3\left(\frac{t}{t_{ff}}\right)^2\ . \tag{15}$$

For any nearly homologous change, this ratio is smallest near the surface so the quasi-hydrostatic approximation is amply justified for $t \gtrsim t_{ff}$. If the lower limit on the time scale derived in §II.C is applied, then $t \gtrsim 150\ t_{ff}$ and $\dot{U}_G/\dot{U}_k \gtrsim 7 \times 10^7$.

c) Approximations for Radiative Transfer

In a stellar interior, the diffusion approximation gives an excellent representation of energy transport by radiation (Schwarzschild 1958, Chapter 2), so this is not normally considered a significant source of error. However, the present investigation is specifically concerned with determining the radius of the Sun, at the photosphere. In this region, the diffusion approximation breaks down and it is necessary to use a different scheme for integrating the structure and locating the photosphere. The calculations described in §III used the Eddington approximation (see Paczynski 1969) for optical depths $\tau < 2/3$, and the diffusion approximation for $\tau \geq 2/3$. We can check this scheme by comparing the atmospheric structure in our equilibrium model with an empirical solar atmosphere, derived from the observed solar spectrum. Figure 6 shows such a comparison with the empirical solar atmosphere

derived by Vernazza, Avrett, and Loeser (1976). The quantities shown are: $T(\tau)$ - temperature vs. optical depth, and $X(\tau)$ - physical depth (with respect to $\tau = 2/3$) vs. optical depth. The latter quantity determines the radius assigned to our models. Our $X(\tau)$ deviates from the empirical relationship by less than 30 km over the primary atmospheric region $0.01 \leq \tau \leq 10$. This should be compared to the radius changes being modeled-- $\Delta R \simeq 6 \times 10^{-4}$ R = 418 km. Thus, the Eddington approximation is sufficiently accurate for our purposes.

d) Mixing Length Theory of Convection

The mixing length theory of convection has been vigorously criticized in the literature. In particular, Gilliland (1980) has argued that uncertainties in the treatment of convection invalidate present attempts to model solar variability. While such criticisms are easily made, the fact remains that a mixing length formalism provides the only presently feasible means of carrying out such calculations. In this case, an assessment of the uncertainties introduced by this approximation is useful. Because there is no "absolute" theory to compare with, this assessment will necessarily be qualitative. It is convenient to consider first the uncertainties in the equilibrium structure and then the uncertainties due to time-dependent effects.

Any reasonable convection theory will predict that the temperature gradient in the bulk of the convection zone is nearly adiabatic. As emphasized by Gough and Weiss (1976), the adiabat is determined

by the requirements that the base of the envelope join smoothly to a radiative interior with a nuclear luminosity of 1 $L_\odot$ and that the radius of the model be 1 $R_\odot$. Our equilibrium models match these parameters to better than 1 part in 10^3, and this fixes the mean adiabat with very little dependence on the convection theory.

In a local mixing length theory, significant deviations from the adiabatic gradient are found in a shallow transition layer, with a depth of $\sim 10^3$ km. The superadiabatic structure of this layer is determined by radiative losses from the convecting elements. Since different mixing length theories assume different bubble geometries, the radiative losses and the structure of the transition layer depend on the specific form of the mixing length theory being used. These specific forms are more or less arbitrary, so this introduces an uncertainty in the equilibrium structure (near the transition layer). As shown by Dearborn and Blake (1980), the direct effect on energy transport due to an α-perturbation is restricted to the transition layer so it would appear that uncertainties in the structure of this layer could translate into a major uncertainty in calculating W for an α-perturbation.

To estimate this uncertainty, we calculated the response of a solar model to an α-perturbation using two fairly different mixing length theories. As a standard theory, we used the formulation of Böhm-Vitense (1958), as modified by Henyey, Vardya, and Bodenheimer (1965). This formulation assumes that a bubble moves a distance (the mixing length) equal to its own diameter. The mixing length is assumed to be proportional to the local pressure scale height. As an alternative model, we used the

formulation described by Chan, Wolff, and Sofia (1981). In this case, the mixing length is chosen to be proportional to the density scale height in the bubble and the bubble geometry is chosen to maximize the convective flux. Both theories contain an arbitrary length scale which must be calibrated by matching the solar radius and luminosity, as described earlier. Figure 7 shows the predicted departures from the adiabatic gradient in the solar transition layer. Note that the degree of superadiabaticity in the two models is very different, though the depth of the transition layers are nearly identical. The latter agreement is a direct result of choosing mixing lengths to match the observed solar radius. The responses of these models to a single α-perturbation were shown earlier, in Figure 2. The calculated values of W differ by only ~20%. This close agreement is a direct result of the calibration procedure, which fixes the depth of the transition layer. We conclude that, because of the observational constraints on the structure of the convection zone, uncertainties in the details of local mixing length theories do not seriously affect the calculated W.

The basic reason that W is so small for an α-perturbation is that the radius change results from expansion of the transition layer and atmosphere and these regions are very shallow. <u>Nonlocal</u> mixing length theories may require a superadiabatic gradient in a boundary layer at the <u>bottom</u> of the convection zone. An expansion of this layer would lead to a large radius increase since the entire convection zone would be lifted and expanded (due to the increase in the hydrostatic pressure scale height). However, any departure from the adiabatic gradient at the bottom of the convec-

tion zone would decrease the total depth of the convection zone. Local mixing length theories predict a total depth of ~165,000 km. This is slightly less than the minimum depth of 175,000 km obtained by Rhodes, Ulrich, and Simon (1977) from an analysis of the solar 5-minute oscillations. A substantial superadiabatic gradient at the bottom of the convection zone would increase this discrepancy.

Finally, none of the convection theories considered above include any dynamical effects. Such effects may alter the response of the model to a perturbation if the perturbation occurs on a time scale shorter than or comparable to the convective turnover time. The observed turnover times at the photosphere vary from ~7 minutes for granulation to ~1 day for supergranulation. Deep in the convective envelope, turnover times may be on the order of 1 month, but we have already noted that this region is nearly adiabatic and difficult to perturb by changes in energy transport. While it is not possible to rule out nonlocal or time-dependent (dynamical) effects, it appears unlikely that such effects would lead to the factor of ten or more increase in $|W|$ required to bring the models into agreement with the observations.

e) Code Dependence

The published values of W for α-perturbations vary by roughly a factor of 10, which is comparable to the discrepancy between the models and the observations. However, with the exception of the incorrect value obtained by Sofia et al. (1979), all of the predicted values of $|W|$ are much smaller than the allowed

observational range. Numerical experiments show that the value of $|W|$ is quite sensitive to the input physics (opacities, for instance) and details of the codes such as interpolation procedures. However, while many of these changes produce values of W smaller than those shown in Figure 2, no reasonable changes produce values substantially larger. This is because, as the predicted radius change for a given α-perturbation becomes larger, it also becomes less sensitive to details of the computing method. The predicted luminosity change is very insensitive to details of input physics and coding.

V. CONCLUSIONS

The solar radius changes found from eclipse observations have the properties

$$\Delta R/R \simeq 6 \times 10^{-4} \ , \ |\Delta L/L| \lesssim 5 \times 10^{-3}$$

and occur on some time scale between 5 days and 50 years. The limit on the ratio of radius to luminosity change is $|W| \gtrsim 0.1$. With these characteristics, the radius changes cannot be due to fluctuations in energy transport through the shallow superadiabatic layers of the solar convective envelope. Sofia and Chan (1981) have examined the response of a solar model to perturbations in total pressure due, for instance, to changing magnetic fields. Again, they found very small ($\sim 10^{-3}$) values of W if the perturbation is confined to shallow layers. These results can be understood by noting that a perturbation at some depth in the envelope affects primarily the layers at and above that depth. The densities and pressures below the perturbed layer are too great for these lower layers to be moved any appreciable distance. Therefore, any mechanism which is confined to shallow layers will produce very small radius changes. We conclude that the observed radius changes are large enough that they must be due to mechanisms which operate at substantial depths below the transition layer of the convective envelope. Sofia and Chan (1981) found $W \simeq 0.1$ for pressure perturbations at depths corresponding to $(1 - M_r/M_\odot) \gtrsim 10^{-4}$. In our models, this corresponds to physical depths $X \gtrsim 27{,}000$ km.

At present, the subject of variations in global solar structure is rich in theoretical speculation but poor in observational data. Further progress will require more extensive and better observations. Simultaneous detection of radius and luminosity variations would replace the present lower limit on $|W|$ with a definite value (or range of values). It may well be true that W is determined by the depth at which the changes originate but is largely independent of the specific physical mechanism. In this case, additional information (such as time scales, periodicities, and possible correlations with solar activity) will be required to pin down the physical mechanisms. Until such observations become available, our understanding of the forces which affect solar structure on human time scales will remain vague.

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REFERENCES

Barnes, J. A., Sargent, H. H., III, and Tryon, P. V. 1980, in *The Ancient Sun*, ed. R. O. Pepin, J. A. Eddy, and R. B. Merrill (New York: Pergamon), p. 159.

Böhm-Vitense, E. 1958, *Zs. f. Ap.*, **46**, 108.

Budyko, M. I. 1969, *Tellus*, **21**, 611.

Chan, K. L., Wolff, C. L., and Sofia, S. 1981, *Ap. J.*, **244**, 582.

Claverie, A., Isaak, G. R., McLeod, C. P., van der Raay, H. B., and Roca Cortes, T. 1979, *Nature*, **282**, 591.

Cox, J. P., and Giuli, R. T. 1968, *Principles of Stellar Structure* (New York: Gordon and Breach).

Dearborn, D. S. P., and Blake, J. B. 1980, *Ap. J.*, **237**, 616.

Dearborn, D. S., and Newman, M. 1978, *Science*, **201**, 150.

Dicke, R. H. 1976, *Solar Phys.*, **47**, 475.

Dicke, R. H., and Goldenberg, H. M. 1967, *Phys. Rev. Letters*, **18**, 313.

Dilke, F. W. W., and Gough, D. O. 1972, *Nature*, **240**, 262.

Dunham, D. W., Sofia, S., Fiala, A. D., Herald, D., and Muller, P. M. 1980, *Science*, **210**, 1243.

Dunham, D. W., Dunham, J. B., Fiala, A. D., and Sofia, S. 1981, in *Workshop on Variations of the Solar Constant*, ed. S. Sofia, in press.

Eddy, J. A., and Boornazian, A. A. 1979, *Bull. A.A.S.*, **11**, 437.

Endal, A. S., and Sofia, S., 1981, *Ap. J.*, **243**, 625.

Fröhlich, C. 1977, in *The Solar Output and its Variations*, ed. O. R. White (Boulder: Colorado Associated University Press), p. 93.

Fröhlich, C., and Brusa, R. W. 1981, Solar Phys., in press.

Gilliland, R. L. 1980, Nature, **286**, 838.

________. 1981, Ap. J., **248**, 1144.

Gough, D. O., and Weiss, N. O. 1976, M.N.R.A.S., **176**, 589.

Henyey, L., Vardya, M. S., and Bodenheimer, P. 1965, Ap. J., **142**, 841.

Hickey, J. R., Stowe, L. L., Jacobowitz, H., Pellegrino, P., Maschoff, R. H., House, F., and Vonder Haar, T. H. 1980, Science, **208**, 281.

Hill, H. A., and Stebbins, R. T. 1975, Ap. J., **200**, 471.

Howard, R., and LaBonte, B. J. 1980, Ap. J., **239**, L33.

Hoyt, D. V. 1979, Rev. Geophys. Space Phys., **17**, 427.

Kondratyev, K. Ya., and Nikolsky, G. A. 1970, Quart. J. Roy. Met. Soc., **96**, 509.

Oster, L. F., Schatten, K. H., and Sofia, S. 1981, Ap. J., in press.

Paczynski, B. 1969, Acta Astron., **19**, 1.

Rhodes, E. J., Jr., Ulrich, R. K., and Simon, G. W. 1977, Ap. J., **218**, 901.

Roosen, R. G., Angione, R. J., and Klemcke, C. H. 1973, Bull. Am. Met. Soc., **54**, 307.

Schwarzschild, M. 1958, Structure and Evolution of the Stars (Princeton: Princeton University Press).

Severny, A. B., Kotov, V. A., and Tsap, T. T. 1976, Nature, **259**, 87.

Shapiro, I. I. 1980, Science, **208**, 51.

Sofia, S., and Chan, K. L. 1981, Solar Phys., in press.

Sofia, S., O'Keefe, J., Lesh, J. R., and Endal, A. S. 1979, Science, **204**, 1306.

Spiegel, E. A., and Weiss, N. O. 1980, Nature, 287, 616.

Sweigart, A. V. 1981, in Workshop on Variations of the Solar Constant, ed. S. Sofia, in press.

Thomas, J. H. 1979, Nature, 280, 663.

Twigg, L. W., and Endal, A. S. 1981, in Workshop on Variations of the Solar Constant, ed. S. Sofia, in press.

Ulrich, R. K. 1975, Science, 190, 619.

Vernazza, J. E., Avrett, E. H., and Loeser, R. 1976, Ap. J. Suppl., 30, 1.

Willson, R. C., Gulkis, S., Janssen, M., Hudson, H. S., and Chapman, G. A. 1981, Science, 211, 700.

Wolff, C. L. 1976, Ap. J., 205, 612.

TABLE 1

SOLAR ECLIPSE DETERMINATIONS OF THE SOLAR RADIUS[a]

Eclipse date	$(\Delta R/R) \times 10^4$	Reference
3 May 1715	$+5.4 \pm 2.1$[b]	Dunham *et al.* (1980)
24 Jan. 1925	$+6.2 \pm 0.8$[c]	Dunham *et al.* (1981)
23 Oct. 1976	-2.4 ± 1.5[c]	Dunham *et al.* (1980)
26 Feb. 1979	-0.8 ± 0.9[c]	Dunham *et al.* (1980)
16 Feb. 1980	-0.3 ± 0.4[c]	Dunham *et al.* (1981)

[a]The quoted changes are measured with respect to a reference radius used in the data analysis.

[b]maximum uncertainty (not 1σ).

[c]1σ uncertainty.

FIGURE CAPTIONS

FIG.1. -- The diagonal line shows the expected expansion/contraction velocities due to radius changes, as a function of the time scale t. The intersection of this line with the spectroscopic detection limit (horizontal line) sets a lower limit on the time scale of the radius changes. The upper limit is based on the interval over which radius changes have been detected. The symbols ($\wedge$) indicate the approximate positions in this diagram of solar pulsations with periods of 5 minutes (cf. Claverie *et al*. 1979) and 160 minutes (cf. Severny *et al*. 1976).

FIG.2. -- The value of W for single α-perturbations, as a function of perturbation amplitude. Solid line and heavy dots--standard mixing length theory; dashed line and triangles--alternative mixing length theory (see §IV.d).

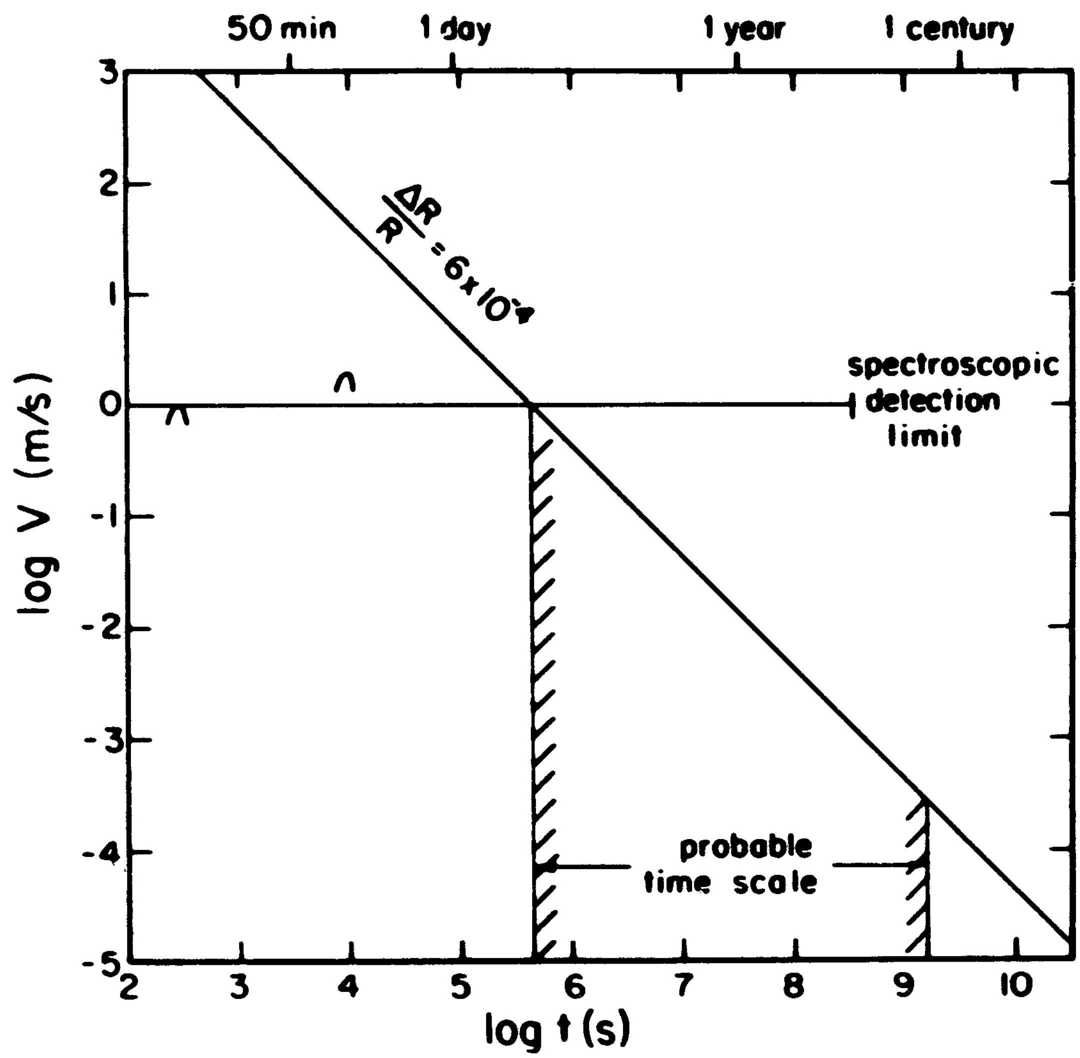
FIG.3. -- Percentage deviation of α from the equilibrium value in the 22-year solar cycle simulation.

FIG.4. -- Changes in the models resulting from the 22-year α-perturbations (see Fig. 3). The curves refer to deviations of R and L from the equilibrium values.

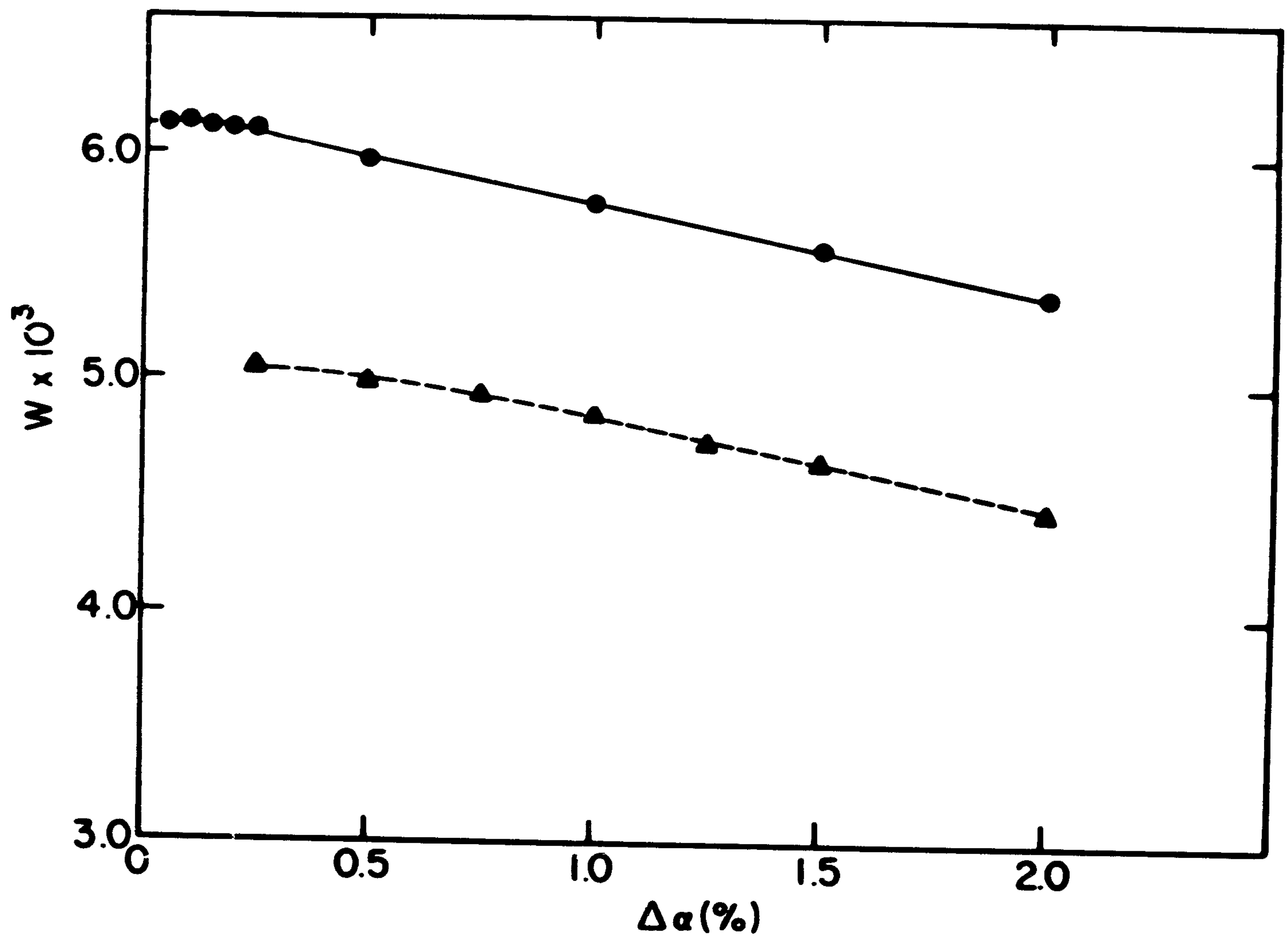
FIG.5. -- Same as Fig. 4, but with $\delta(R)$ and $\delta(L)$ referring to changes in models separated by an 11-year interval.

FIG.6. -- The $T(\tau)$ and $X(\tau)$ relationships for a solar atmosphere based on the Eddington approximation (solid lines) and an empirical solar atmosphere (dashed lines).

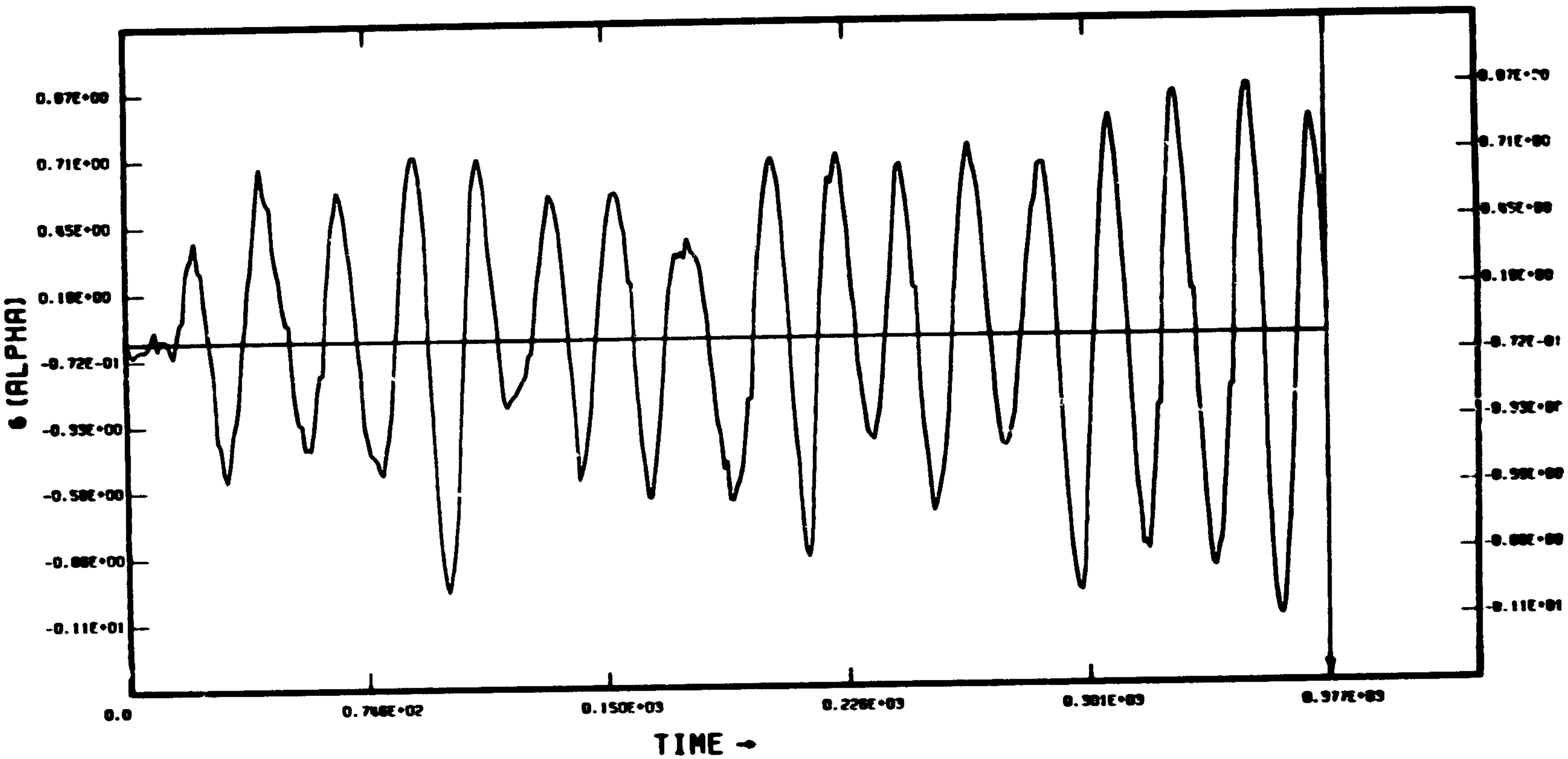
FIG.7. -- Difference between the actual temperature gradient $\nabla = (d\ell nT/d\ell nP)$ and the adiabatic gradient $\nabla_{ad} = (\partial \ell nT/\partial \ell nP)_{ad}$ as a function of depth below the photosphere ($\tau = 2/3$). The solid line shows an equilibrium solar model computed with the Böhm-Vitense (1958) mixing length theory and the dashed line shows the same model computed with the Chan et al. (1981) mixing length theory.



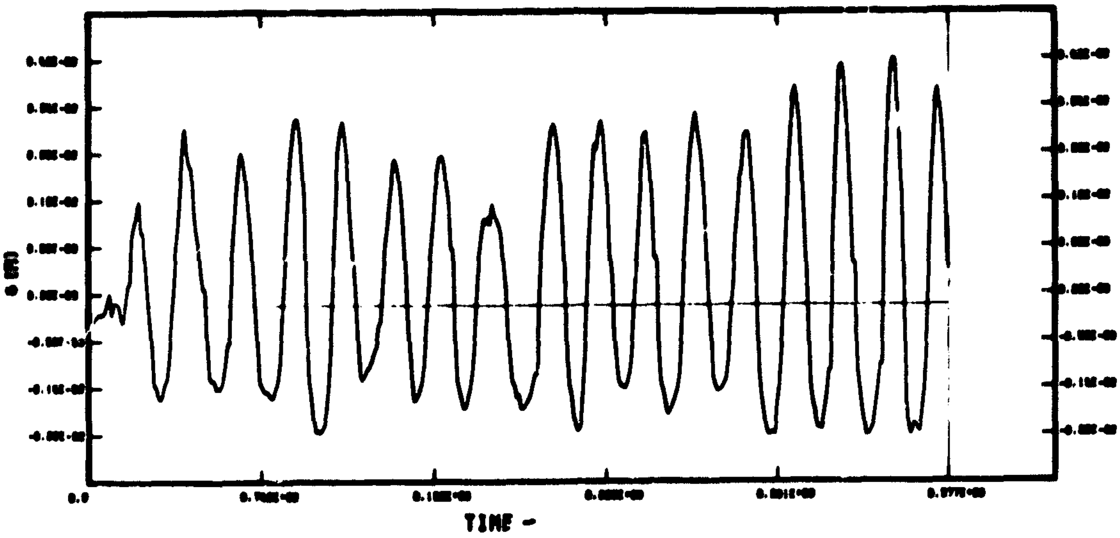
50 min
1 day
1 year
1 century
$\frac{\Delta R}{R} = 6 \times 10^{-4}$
spectroscopic
detection
limit
probable
time scale
log V (m/s)
log t (s)
3
2
1
0
-1
-2
-3
-4
-5
2
3
4
5
6
7
8
9
10

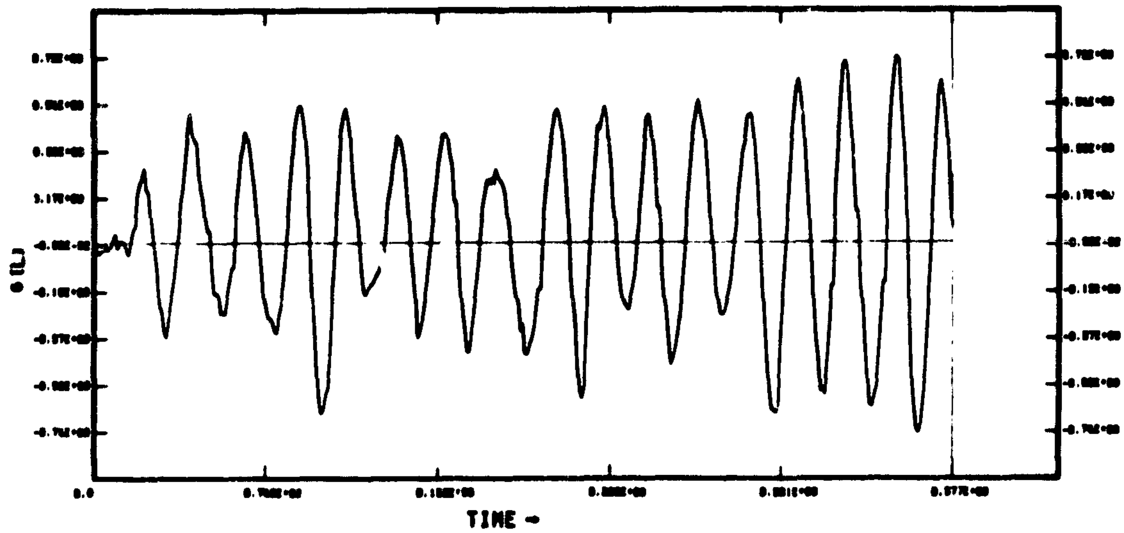


6.0
5.0
4.0
3.0
W x 10^3
0
0.5
1.0
1.5
2.0
Δα(%)



0.87E+00
0.71E+00
0.65E+00
0.10E+00
-0.72E-01
-0.33E+00
-0.58E+00
-0.88E+00
-0.11E+01
6 (ALPHA)
0.0
0.760E+02
0.150E+03
0.228E+03
0.301E+03
0.377E+03
TIME →


TIME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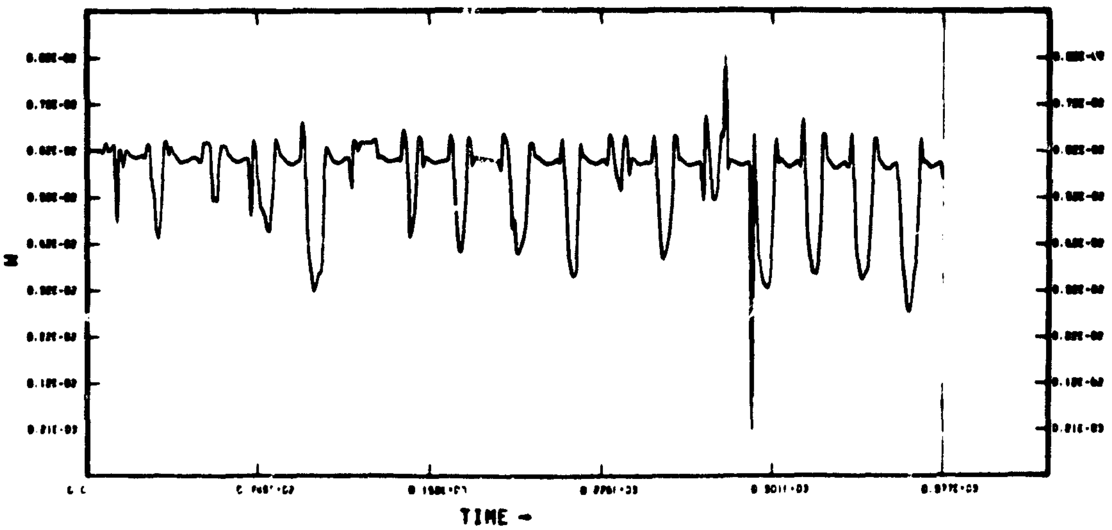
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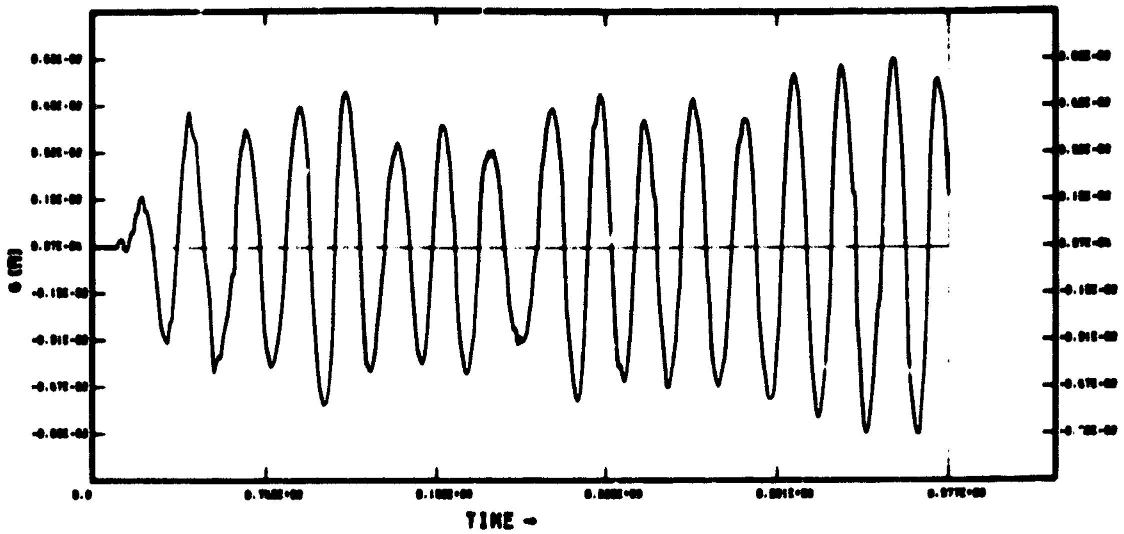
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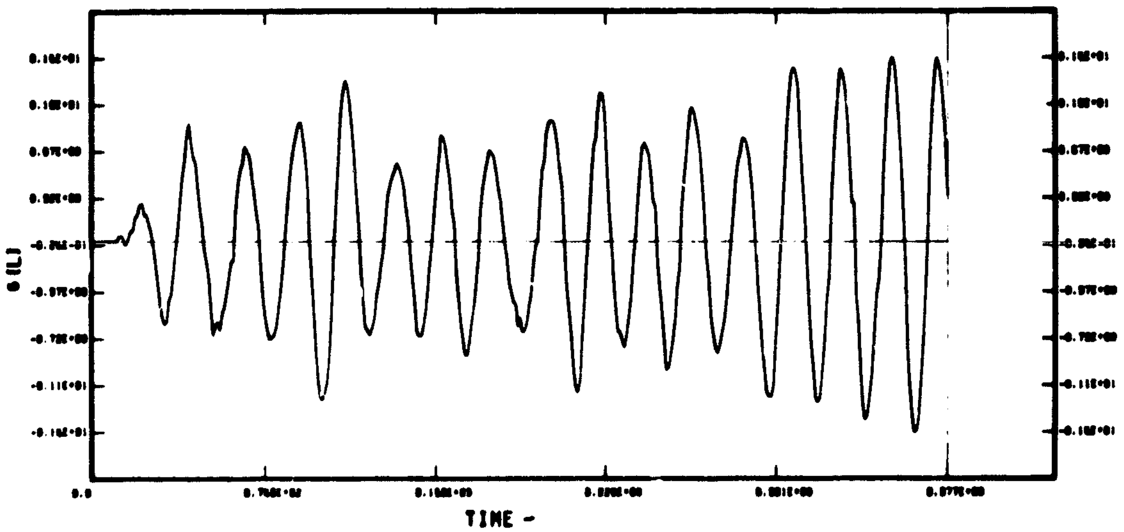
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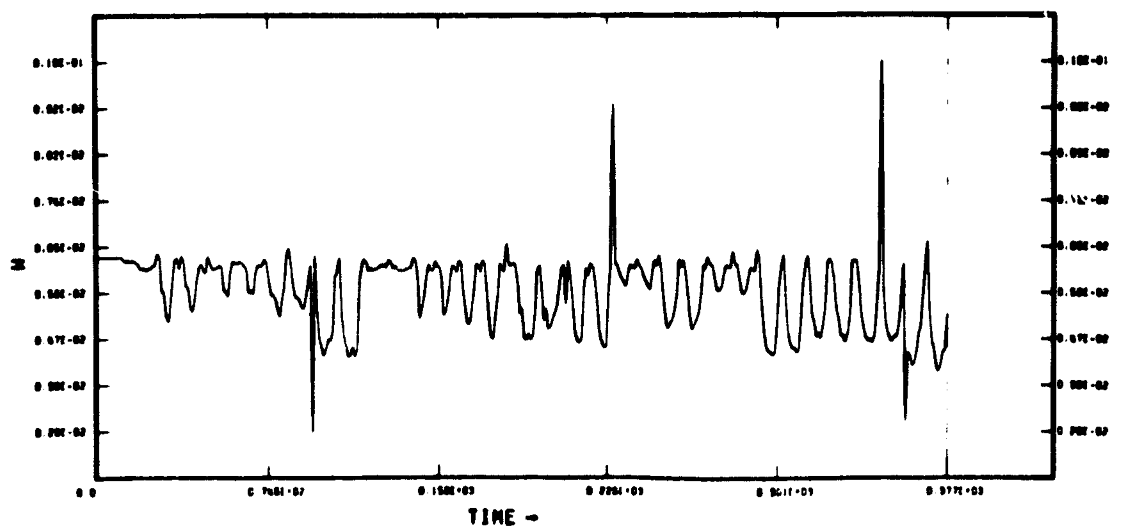
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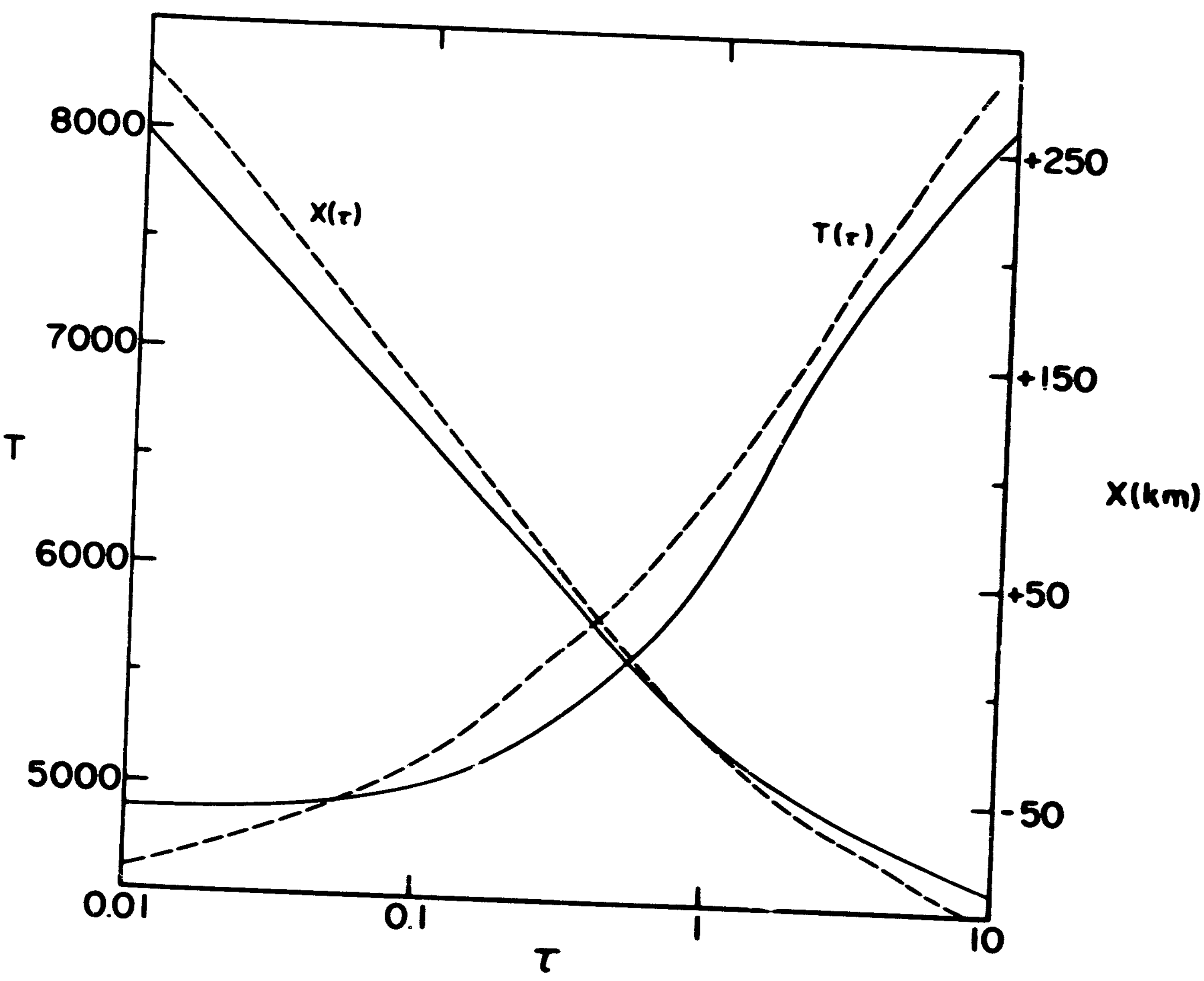
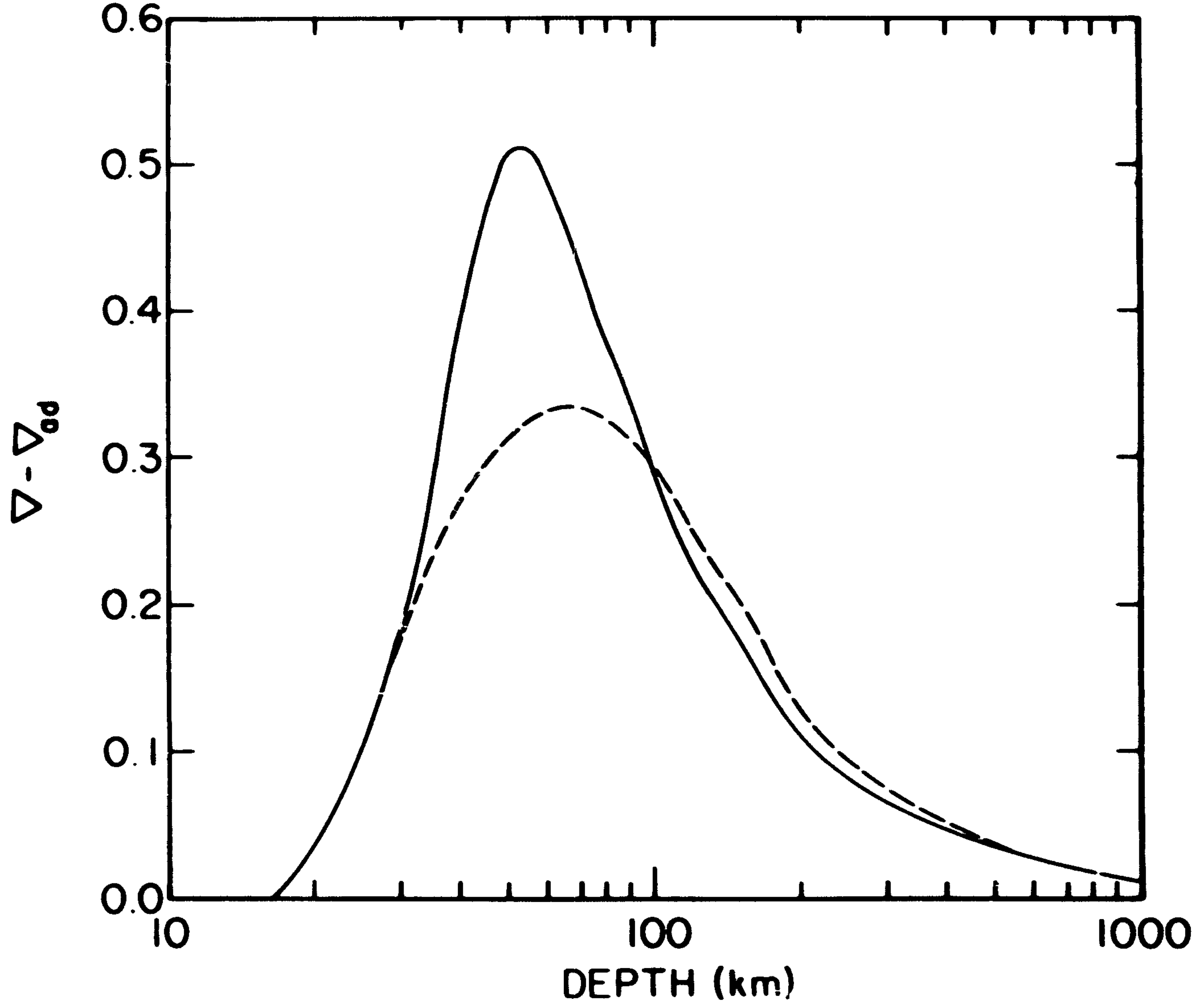


8000
7000
6000
5000
T
X(τ)
T(τ)
+250
+150
+50
-50
X(km)
0.01
0.1
1
10
τ



$\nabla - \nabla_{ad}$
0.6
0.5
0.4
0.3
0.2
0.1
0.0
10
100
1000
DEPTH (km)